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THE

MATHEMATICAL MONTHLY.

OCTOBER, 1858.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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CONTENTS.

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INTRODUCTORY NOTE.

THE attempt to establish a Mathematical Journal is a step of too great importance to be taken without due deliberation, — without carefully considering the end to be attained, and the means to be employed in securing it. This end may be either the *advancement* of the science, or the *elevation of the standard* of mathematical learning. Now it is not probable that a journal of a high scientific character, having the former end solely in view, could be sustained, for it would contain only one element of interest, and that one for the few professed mathematicians.

But a journal having the latter end in view, if successful in the highest sense, must necessarily to a greater or less degree involve the former; and the question arises, May not such a journal have a scope sufficiently comprehensive and elastic to embrace all grades of talent and attainment, and, therefore, corresponding elements of interest? If so, then it should embrace students in one extreme, and professed mathematicians in the other; which extremes neces-

sarily include all intermediate grades of teachers and laborers in this vast field.

Should it be a journal merely of problems and solutions? We think not. It should cover the *whole ground* sketched in the following circular note.

To the student, or younger mathematician, problems are indispensable to make him *sure* of, and *ground* him in, the theory, as well as exercise and develop his skill. But it is a serious question, whether it is not a waste of time and energy to put scores of able mathematicians upon the same problem, any one of whom has the knowledge and skill necessary for the complete solution. The problems should, therefore, usually be selected with reference to the learner, and so graduated as to suit different degrees of knowledge and skill, even in the same branch.

Problems of the highest grade, especially if they are likely to lead the investigator into a comparatively new field, or develop methods or important practical results, may occasionally be published as *challenges;* but generally, we think it advisable to publish the solution of such problems at once, and if those particularly interested in the solution should be led to any new and curious developments, insert them afterwards.

The Journal should contain, for all learners, clear and concise notes upon all points of theory and application in all branches of the science; and these notes should come from able contributors, who can be plain without being weak,—who can unite simplicity of treatment with elegance of style.

It should contain "all scraps of mathematical writing too good to be lost," whether *elementary* or *profound*, whether original in *manner* or *matter*, whether complete in themselves or to be resumed at the convenience of the author, whether notices or reviews of matter old or new; in short, every thing fitly designated by "*notes and queries.*"

Besides, it should contain "carefully elaborated essays, chiefly valuable perhaps as promises of better things hereafter," as well as those of a higher character.

There is another large field in which the Journal will find its legitimate work,—one in which it can do the double duty of inducting students and younger mathematicians into the highest departments of the science, and of opening to the abler and more experienced an opportunity to contribute their share to the noble work of elevating the standard of mathematical learning in the country. All mathematicians know that there are many subjects in the higher departments of the science upon which little, if any thing, has as yet been written among us. Now, if they will take these subjects and develop them fully and systematically through the pages of the Journal, they may afterwards be issued in a separate form from the stereotype plates, at a very small cost. In such cases the right and benefit thereof shall vest in the author.

In this way we shall secure the coöperation of all: of students and younger mathematicians, for the range embraces them with their respective abilities and attainments, and therefore interests; of professed mathematicians, for, besides the large field specified above, neither their dignity nor scientific character can be affected by communicating notes which might not be of sufficient importance to warrant insertion in a Journal of high scientific pretensions.

In fine, then, the Journal will be to the professed mathematician a *recreation* and a *study*, while to the student it will be a *study* and an *example*.

Being convinced that a Journal of this character, in which all interests shall blend and coöperate, is needed, that it "will occupy ground unoccupied by other periodicals, and will be of great importance in advancing the intellectual character of our country;" and believing, that, if properly sustained by those who ought to have its

success most deeply at heart, it can be sustained in its financial department, we have taken the liberty during the last few weeks, as a preliminary inquiry, to send copies of the following note. The replies already received are of such a character as to fully warrant its issue in this more general manner. Every one, feeling any interest in this enterprise, is earnestly solicited to express his views fully, and state to what extent, if any, he will coöperate; so that his name may be included in the following list. As this introductory note will be substantially retained in the first number, the names of many of those who will give to the Journal its vitality and usefulness will thus be enrolled together: an array and diversity of talent that will make it a means of culture, which the friends of good learning in our country "will not willingly let die."

CIRCULAR NOTE.

Nautical Almanac Office, Cambridge, February 13, 1858.

Dear Sir, — Allow me to call your attention to the following considerations: You are aware, that, while almost every science, as well as art, has its own appropriate journal, around which corresponding interests and tastes cluster, by which special research is encouraged, and through which all the valuable results are communicated to the world, the science of Mathematics is still without its own particular organ.

Now it seems to us that such a journal is needed; one that shall embrace, among its contributors, the best talent, in order that younger laborers in the same field may always have before them a high standard of excellence, and that it may be a fair index of the mathematical ability of the country. On the

other hand, however, care should be taken not to graduate it, as a whole, too high above the average attainments of mathematical students; otherwise, only the few would be interested in it or benefited by it. It should therefore embrace in its pages solutions, demonstrations, and discussions in all branches of the science, as well as in all its various applications.

It should contain notes and queries, notices and reviews of all the principal mathematical works issued in this country, as well as in Europe.

In short, it should be the medium of all kinds of information pertaining to the science, to which a large proportion of our mathematical students have at present no ready access.

Such is, in brief, our idea of the character the journal should possess to insure to it the greatest usefulness and most permanent success.

This note, with the following queries, will be addressed to many of the most eminent mathematicians and educators in the various parts of the United States; and upon the several replies we shall base our future action.

Do you think there is a present need of a mathematical journal of any kind?

Do your views coincide with those here expressed as to its character? If not, be pleased to state your views.

Are you willing to assist in establishing and sustaining a journal by contributing to its pages?

Will you allow such use to be made of your reply to this note as may be proper to carry the proposed plan into effect?

A *decided* reply is respectfully solicited, whether favorable or otherwise.

With much esteem,

Yours truly,

J. D. Runkle.

To this note the following gentlemen have already replied:—

*Prof. Charles Avery, Hamilton Coll., Clinton, N. Y.
*Dr. A. D. Bache, Sup't U. S. Coast Survey, Washington, D. C.
*Mr. F. W. Bardwell, Naut. Alm. Office, Cambridge, Mass.
Prof. W. H. C. Bartlett, Military Academy, West Point.
*Prof. G. P. Bond, Harvard Coll. Obs., Cambridge, Mass.
Hon. G. S. Boutwell, Sec'y Board of Education, Boston, Mass.
Prof. Francis Bowen, Harvard Coll., Cambridge, Mass.
*Mr. Isaac Bradford, Naut. Alm. Office, Cambridge, Mass.
Dr. F. Brünnow, Director Obs., Ann Arbor, Mich.
Prof. A. E. Church, Military Academy, West Point.
Prof. Charles Davies, Columbia Coll., New York.
*Mr. George Eastwood, Naut. Alm. Office, Cambridge, Mass.
*Prof. H. L. Eustis, Lawrence Scientific School, Cambridge, Mass.
*Prof. W. Ferrel, Nashville, Tenn.
*Dr. B. A. Gould, Jr., Director Dudley Obs., Albany, N. Y.
*Mr. D. B. Hagar, Prin. Eliot High School, Jamaica Plain, Mass.
*Prof. R. D. Hamilton, Hobart Free Coll., Geneva, N. Y.
*Mr. J. B. Henck, Civil Engineer, Boston.
*Mr. W. D. Henkle, Richmond, Indiana.
*Prof. Joseph Henry, Sec'y Smithsonian Inst., Washington, D. C.
*Rev. Thomas Hill, Waltham, Mass.
Prof. E. N. Horsford, Lawrence Scientific School, Cambridge, Mass.
Prof. Elias Loomis, New York University, N. Y.
Prof. D. H. Mahan, Military Academy, West Point.
Prof. O. M. Mitchell, Director Obs., Cincinnati, Ohio.
*Mr. Simon Newcomb, Naut. Alm. Office, Cambridge, Mass.
Prof. H. A. Newton, Yale Coll., New Haven, Conn.
*Mr. James E. Oliver, Naut. Alm. Office, Lynn, Mass.
Prof. J. W. Patterson, Dartmouth Coll., Hanover, N. H.

*Prof. Benjamin Peirce, Harvard Coll., Cambridge, Mass.
Prof. George R. Perkins, Albany, N. Y.
Dr. C. H. F. Peters, Albany, N. Y.
*Prof. O. Root, Hamilton Coll., Clinton, N. Y.
*Mr. Truman Henry Safford, Naut. Alm. Office, Cambridge, Mass.
*Mr. Thomas Sherwin, Prin. English High School, Boston.
Prof. E. S. Snell, Amherst Coll., Amherst, Mass.
Dr. A. W. Smith, Middletown, Conn.
*Prof. M. C. Stevens, Richmond, Indiana.
Prof. T. Strong, Rutgers Coll., New Brunswick, N. J.
Prof. John Tatlock, Williams Coll., Williamstown, Mass.
Prof. John Towler, Hobart Free Coll., Geneva, N. Y.
*Prof. J. M. Van Vleck, Middletown, Conn.
*Mr. William Watson, Instructor, Lawrence Scientific School.
*Prof. Joseph Winlock, Sup't Am. Naut. Alm., Cambridge, Mass.
Prof. J. S. Woodman, Dartmouth Coll., Hanover, N. H.
*Mr. Chauncey Wright, Naut. Alm. Office, Cambridge, Mass.

We had intended to give such extracts from these replies as would indicate their nature; but are reluctantly compelled, from their number and length, to substitute the following summary.

To the question, "Do you think there is a present need of a mathematical journal of any kind?" the replies are unanimous in the affirmative. To the question, "Do your views coincide with those here expressed as to its character?" the replies are unanimous in the affirmative. To the question, "Are you willing to assist in establishing and sustaining a journal, by contributing to its pages?" the replies, with very few exceptions, promise such aid as may be consistent with other duties; while those whose names are marked with a star have pledged constant and active coöperation.

RUNKLE'S PRIZES

TO THE STUDENTS IN ANY INSTITUTION OF LEARNING IN THE UNITED STATES OR BRITISH PROVINCES.

I. For Solutions.

Judges.

Prof. Joseph Winlock, Mr. Chauncey Wright,
Mr. Truman Henry Safford.

In the first, and each succeeding number of the Journal during the year, we shall publish five problems, entitled *Prize Problems for Students.* The student who shall send us the best solutions of the greatest number of the prize problems, in any number of the Journal, in time for the third number following the one in which they are proposed, shall be entitled to a first prize of *ten dollars;* the second in order of merit shall be entitled to a second prize of *a bound copy of the first volume of the Journal.* Each solution will be estimated by the judges independently, and to the students who have the first and second highest aggregates of marks shall be awarded the prizes. The best solution of each problem will be published, entitled *Prize Solution,* with such other solutions as the judges shall consider of sufficient merit. All the steps in each solution must be fully given, and the whole communicated in a plain and legible handwriting, to secure attention. The award of the judges, announcing the names of the successful competitors, the prize solutions with the names of the authors, and credit for all solutions received, will be published in the fourth and succeeding numbers. No student shall be entitled to the same prize twice during the same year; but full credit will be given him in the award of the judges.

II. For Essays.

Judges.

Prof. W. Ferrel, Mr. J. B. Henck,
Prof. George R. Perkins.

A first prize of *fifty dollars* for the best essay.

A second prize of *forty dollars* for the next in order of merit.

A third prize of *thirty dollars* for the third in order.

A fourth prize of *twenty dollars* for the fourth.

A fifth prize of *ten dollars* for the fifth.

These prizes will be given for essays upon any questions in pure or applied mathematics, including those questions in physics "which can be solved only by an application of mathematical logic to the fundamental principles which constitute the scientific conception of the phenomena."

The essays are not to be simply descriptive, but thoroughly digested discussions of the questions under consideration, presented with clearness and symmetry. Originality in matter or treatment, or in both, is certainly desirable; but even without these, an essay may show such a complete mastery of the subject as to be highly meritorious.

Essays must be brief, not exceeding in length eight pages of the Journal, and must be judged worthy of publication to be entitled to any prize.

They must be received at or before the date of publication of the eighth number, that the award of the judges may be announced, and the essays published in the first volume.

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J. D. Runkle.

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*Mr. A. Agassiz, Cambridge, Mass.
Prof. Stephen Alexander, College of New Jersey, Princeton.
*Prof. James L. Alverson, Genesee Coll., Lima, N. Y.
Prof. Theodore Appel, Franklin and Marshall Coll., Lancaster, Pa.
Prof. Samuel Alsop, West Chester, Penn.
*Prof. Charles Avery, Hamilton Coll., Clinton, N. Y.
*Mr. R. S. Avery, U. S. Coast Survey, Washington, D. C.
*Dr. A. D. Bache, Sup't U. S. Coast Survey, Washington, D. C.
Prof. Mark Bailey, Franklin Coll., Franklin, Ind.
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Major J. G. Barnard, U. S. A., New York.
Prof. W. H. C. Bartlett, Military Academy, West Point.
*Prof. G. P. Bond, Harvard Coll. Obs., Cambridge, Mass.
Hon. G. S. Boutwell, Sec'y Board of Education, Boston, Mass.
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*Prof. W. P. Trowbridge, U. S. Coast Survey, Washington, D. C.

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*Mr. G. B. Vose, U. S. Coast Survey, Washington, D. C.
Prof. P. T. Walters, Wake Forest Coll., N. C.
Mr. Wm. R. Ware, Architect, Boston.
*Mr. William Watson, Instructor, Lawrence Scientific School.
*President N. B. Webster, Va. Collegiate Institute, Portsmouth, Virginia.
Prof. Henry H. White, Kentucky Univ., Harrodsburg.
*Prof. Geo. C. Whitlock, Victoria Coll., Coburg, Canada West.
Lieut. Wm. H. Willcox, U. S. N., Naval Academy, Annapolis, Md.
Prof. J. C. Wills, Randolph-Macon Coll., Virginia.
Prof. William Wines, Stewart Coll., Clarksville, Tenn.
*Prof. Joseph Winlock, Sup't Am. Naut. Alm., Cambridge, Mass.
Prof. Devolson Wood, Univ. of Michigan, Ann Arbor.
Prof. J. S. Woodman, Dartmouth Coll., Hanover, N. H.
*Mr. Chauncey Wright, Naut. Alm. Office, Cambridge, Mass.
Prof. Charles A. Young, Western Reserve Coll., Madison, Ohio.
*Prof. W. H. Young, Ohio Univ., Athens.
Prof. Ira Young, Dartmouth Coll., Hanover, N. H.

We had intended to give such extracts from these replies as would indicate their nature ; but are reluctantly compelled, from their number and length, to substitute the following summary :

To the question, " Do you think there is a present need of a mathematical journal of any kind ? " the replies are unanimous in the affirmative. To the question, " Do your views coincide with those here expressed as to its character ? " the replies are unanimous in the affirmative. To the question, " Are you willing to assist in establishing and sustaining a journal, by contributing to its pages ; the replies, with very few exceptions, promise such aid as may be consistent with other duties ; while those whose names are marked with a star have pledged constant and active coöperation.

The following Report and Resolution were unanimonsly adopted in the Section of Mathematics and Physics, at the meeting of the American Association for the Advancement of Science, held at Baltimore.

" The Committee to whom was referred Mr. Runkle's project for the establishment of a Mathematical Journal have examined the subject, and submit their Report as comprised in the following Resolution, which they recommend to the adoption of the Section.

" *Resolved*, that we deem the establishment of a Mathematical Journal upon the plan proposed by Mr. J. D. Runkle, if well executed, to be highly important to the mathematical progress of the country, and the advancement of science ; and that we have full confidence in Mr. Runkle to do justice to the task which he has undertaken.

" A. Caswell,
Benjamin Peirce,
Geo. W. Coaklay."

"*April*, 30, 1858."

RUNKLE'S PRIZES

TO THE STUDENTS IN ANY INSTITUTION OF LEARNING IN THE UNITED STATES OR BRITISH PROVINCES.

I. For Solutions.

Judges.

Prof. Joseph Winlock, Mr. Chauncey Wright,
Mr. Truman Henry Safford.

In the first and each succeeding number of the Journal during the year, we shall publish five problems, entitled *Prize Problems for Students.* The student who shall send us the best solutions of the greatest number of the prize problems, in any number of the Journal, in time for the third number following the one in which they are proposed, shall be entitled to a first prize of *ten dollars;* the second in order of merit shall be entitled to a second prize of *a bound copy of the first volume of the Journal.* Each solution will be estimated by the judges independently, and to the students who have the first and second highest aggregates of marks shall be awarded the prizes. The best solution of each problem will be published, entitled *Prize Solution,* with such other solutions as the judges shall consider of sufficient merit. All the steps in each solution must be fully given, and the whole communicated in a plain and legible handwriting, to secure attention. The award of the judges, announcing the names of the successful competitors, the prize solutions with the names of the authors, and credit for all solutions received, will be published in the fourth and succeeding numbers. No student shall be entitled to the same prize twice during the same year; but full credit will be given him in the award of the judges.

II. For Essays.

Judges.

Prof. W. Ferrel, Mr. J. B. Henck,
Prof. George R. Perkins.

A first prize of *fifty dollars* for the best essay.

A second prize of *forty dollars* for the next in order of merit.

A third prize of *thirty dollars* for the third in order.

A fourth prize of *twenty dollars* for the fourth.

A fifth prize of *ten dollars* for the fifth.

These prizes will be given for essays upon any questions in pure or applied mathematics, including those questions in physics "which can be solved only by an application of mathematical logic to the fundamental principles which constitute the scientific conception of the phenomena."

The essays are not to be simply descriptive, but thoroughly digested discussions of the questions under consideration, presented with clearness and symmetry. Originality in matter or treatment, or in both, is certainly desirable; but even without these, an essay may show such a complete mastery of the subject as to be highly meritorious.

Essays must be brief, not exceeding in length eight pages of the Journal, and must be judged worthy of publication to be entitled to any prize.

They must be received at or before the date of publication of the eighth number, that the award of the judges may be announced, and the essays published in the first volume.

NOTE.

We cannot omit this opportunity to express our sincere thanks to those who have already given this enterprise their attention and approval; and we think we hazard nothing in saying, that the interest and coöperation it has secured, and is daily securing, must insure its complete success. In calling attention to our Publisher's Prospectus we would say, that we cannot ask him to assume a responsibility in an enterprise in which he will have no direct pecuniary interest; nor is it possible for us to assume more than our pledges already involve. For those who have taken a personal interest in this matter, by sending us their approval of the undertaking, we feel that not a single word is necessary. We ask every one who may receive this circular to give the subject his serious attention, and if it meets his approval, we beg him to remember, that by his subscription and influence he can assist in establishing a journal that cannot fail, with the ability pledged to sustain it, to do a real and lasting good, not only to the educational interests of the country, but to all the exact sciences.

We should not venture to make this positive appeal, if this enterprise were any longer a personal matter; but it is only necessary to look at the list of names to perceive that it is a matter of much more general interest. Finally, we ask it as a personal favor of all to act promptly, that the publication of the Journal may be commenced as early as possible.

PROSPECTUS

OF THE

MATHEMATICAL MONTHLY.

Each number of the Mathematical Monthly will contain at least twenty-four pages, in the same type, page, and paper as the " Introductory Note."

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JOHN BARTLETT, *Publisher.*

Cambridge, April 21, 1858.

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THE

MATHEMATICAL MONTHLY.

INTRODUCTORY NOTE.

The attempt to establish a Mathematical Journal is a step of too great importance to be taken without due deliberation, — without carefully considering the end to be attained, and the means to be employed in securing it. This end may be either the *advancement* of the science, or the *elevation of the standard*, of mathematical learning. Now it is not probable that a journal of a high scientific character, having the former end solely in view, could be sustained, for it would contain only one element of interest, and that one for the few professed mathematicians.

But a journal having the latter end in view, if successful in the highest sense, must necessarily to a greater or less degree involve the former; and the question arises, May not such a journal have a scope sufficiently comprehensive and elastic to embrace all grades of talent and attainment, and, therefore, corresponding elements of interest? If so, then it should embrace students in one extreme, and professed mathematicians in the other; which extremes neces-

sarily include all intermediate grades of teachers and laborers in this vast field.

Should it be a journal merely of problems and solutions? We think not. It should cover the *whole ground* sketched in the following circular note.

To the student, or younger mathematician, problems are indispensable to make him *sure* of, and *ground* him in, the theory, as well as exercise and develop his skill. But it is a serious question, whether it is not a waste of time and energy to put scores of able mathematicians upon the same problem, any one of whom has the knowledge and skill necessary for the complete solution. The problems should, therefore, usually be selected with reference to the learner, and so graduated as to suit different degrees of knowledge and skill even in the same branch.

Problems of the highest grade, especially if they are likely to lead the investigator into a comparatively new field, or develop methods or important practical results, may occasionally be published as *challenges;* but generally, we think it advisable to publish the solution of such problems at once, and if those particularly interested in the solution should be led to any new and curious developments, insert them afterwards.

The Journal should contain, for all learners, clear and concise notes upon all points of theory and application in all branches of the science; and these notes should come from able contributors, who can be plain without being weak,— who can unite simplicity of treatment with elegance of style.

It should contain "all scraps of mathematical writing too good to be lost," whether *elementary* or *profound,* whether original in *manner* or *matter,* whether complete in themselves or to be resumed at the convenience of the author, whether notices or reviews of matter old or new; in short, every thing fitly designated by "*notes and queries.*"

Besides, it should contain "carefully elaborated essays, chiefly valuable perhaps as promises of better things hereafter," as well as those of a higher character.

There is another large field in which the Journal will find its legitimate work, — one in which it can do the double duty of inducting students and younger mathematicians into the highest departments of the science, and of opening to the abler and more experienced an opportunity to contribute their share to the noble work of elevating the standard of mathematical learning in the country. All mathematicians know that there are many subjects in the higher departments of the science upon which little, if any thing, has as yet been written among us. Now, if they will take these subjects and develop them fully and systematically through the pages of the Journal, they may afterwards be issued in a separate form from the stereotype plates, at a very small cost. In such cases the right and benefit thereof shall vest in the author.

In this way we shall secure the coöperation of all: of students and younger mathematicians, for the range embraces them with their respective abilities and attainments, and therefore interests; of professed mathematicians, for, besides the large field specified above, neither their dignity nor scientific character can be affected by communicating notes which might not be of sufficient importance to warrant insertion in a Journal of high scientific pretensions.

In fine, then, the Journal will be to the professed mathematician a *recreation* and a *study*, while to the student it will be a *study* and an *example*.

Being convinced that a Journal of this character, in which all interests shall blend and coöperate, is needed, that it "will occupy ground unoccupied by other periodicals, and will be of great importance in advancing the intellectual character of our country;" and believing, that, if properly sustained by those who ought to have its

success most deeply at heart, it can be sustained in its financial department, we have taken the liberty during the last few weeks, as a preliminary inquiry, to send copies of the following note. The replies already received are of such a character as to fully warrant its issue in this more general manner. Every one, feeling any interest in this enterprise, is earnestly solicited to express his views fully, and state to what extent, if any, he will coöperate; so that his name may be included in the following list. As this introductory note will be substantially retained in the first number, the names of many of those who will give to the Journal its vitality and usefulness will thus be enrolled together: an array and diversity of talent that will make it a means of culture, which the friends of good learning in our country "will not willingly let die."

CIRCULAR NOTE.

NAUTICAL ALMANAC OFFICE, CAMBRIDGE, February 13, 1858.

DEAR SIR, — Allow me to call your attention to the following considerations: You are aware, that, while almost every science, as well as art, has its own appropriate journal, around which corresponding interests and tastes cluster, by which special research is encouraged, and through which all the valuable results are communicated to the world, the science of Mathematics is still without its own particular organ.

Now it seems to us that such a journal is needed; one that shall embrace, among its contributors, the best talent, in order that younger laborers in the same field may always have before them a high standard of excellence, and that it may be a fair index of the mathematical ability of the country. On the other hand, however, care should be taken not to graduate it, as a whole, too high above the average attainments of mathematical students; otherwise, only the few would be interested in it or benefited by it. It should therefore embrace in its pages solutions, demonstrations, and discussions in all branches of the science, as well as in all its various applications.

It should contain notes and queries, notices and reviews of all the principal mathematical works issued in this country, as well as in Europe.

In short, it should be the medium of all kinds of information pertaining to the science, to which a large proportion of our mathematical students have at present no ready access.

Such is, in brief, our idea of the character the journal should possess to insure to it the greatest usefulness and most permanent success.

This note, with the following queries, will be addressed to many of the most eminent mathematicians and educators in the various parts of the United States; and upon the several replies we shall base our future action.

Do you think there is a present need of a mathematical journal of any kind?

Do your views coincide with those here expressed as to its character? If not, be pleased to state your views.

Are you willing to assist in establishing and sustaining a journal by contributing to its pages?

Will you allow such use to be made of your reply to this note as may be proper to carry the proposed plan into effect?

A *decided* reply is respectfully solicited, whether favorable or otherwise.

With much esteem,

Yours truly,

J. D. Runkle.

To this note the following gentlemen have already replied:—

*Mr. A. Agassiz, Cambridge, Mass.
Prof. Stephen Alexander, College of New Jersey, Princeton.
*Prof. James L. Alverson, Genesee Coll., Lima, N. Y.
Prof. Theodore Appel, Franklin and Marshall Coll., Lancaster, Pa.
Prof. Samuel Alsop, West Chester, Penn.
Prof. John Aubier, St. John's Coll., Fordham, N. Y.
*Prof. Charles Avery, Hamilton Coll., Clinton, N. Y.
*Mr. R. S. Avery, U. S. Coast Survey, Washington, D. C.
*Dr. A. D. Bache, Sup't U. S. Coast Survey, Washington, D. C.
Prof. Mark Bailey, Franklin Coll., Franklin, Ind.
*Mr. F. W. Bardwell, Naut. Alm. Office, Cambridge, Mass.
*President F. A. P. Barnard, Univ. of Miss., Oxford.
Hon. Henry Barnard, Hartford, Conn.
Major J. G. Barnard, U. S. A., New York.
Prof. W. H. C. Bartlett, Military Academy, West Point.
*Prof. A. T. Bledsoe, University of Virginia.
*Prof. G. P. Bond, Harvard Coll. Obs., Cambridge, Mass.
Hon. G. S. Boutwell, Sec'y Board of Education, Boston, Mass.
Mr. J. Ingersoll Bowditch, Boston, Mass.
Prof. Francis Bowen, Harvard Coll., Cambridge, Mass.
*Mr. Isaac Bradford, Naut. Alm. Office, Cambridge, Mass.
Prof. John Brocklesby, Trinity Coll., Hartford, Conn.
Prof. P. P. Brown, Madison Univ., Hamilton, N. Y.
President John H. Bruner, Hiwasse Coll., Monroe Co., Tenn.
Dr. F. Brünnow, Director Obs., Ann Arbor, Mich.
Prof. John L. Campbell, Wabash Coll., Crawfordsville, Indiana.
Prof. David J. Capron, St. John's Coll., Annapolis, Md.
Prof. Alexis Caswell, Brown Univ., Providence, R. I.
*Prof. Wm. Chauvenet, Naval Academy, Annapolis, Md.

*Prof. J. B. CHERRIMAN, University Coll., Toronto, Canada West.
Prof. A. E. CHURCH, Military Academy, West Point.
Prof. JAMES CLARK, Georgetown Coll., D. C.
Prof. JOHN E. CLARK, Univ. of Michigan, Ann Arbor.
*Prof. A. W. CLARKE, Washington Coll., Chestertown, Md.
*Prof. GEO. W. COAKLAY, St. James' Coll., Washington Co., Md.
*Prof. J. H. C. COFFIN, Naval Academy, Annapolis, Md.
Prof. JAMES H. COFFIN, LaFayette Coll., Easton, Pa.
Prof. J. H. COIT, St. James' Coll., Washington Co., Md.
Mr. DANA P. COLBURN, Prin. Normal School, Bristol, R. I.
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President N. M. CRAWFORD, Mercer Univ., Penfield, Georgia.
Prof. JAMES D. DANA, Yale Coll., New Haven, Conn.
Prof. CHARLES DAVIES, Columbia Coll., New York.
Prof. S. S. DOAK, Hiwassee Coll., Monroe Co. Tenn.
Prof. P. W. DODSON, Union Univ., Murfreesboro, Tenn.
Prof. F. B. DOWNES, Homer Academy, Homer, N. Y.
*Mr. JOHN DOWNES, United States Coast Survey, Washington, D. C.
Prof. JOHN T. DUFFIELD, Coll. of New Jersey, Princeton.
*Mr. GEORGE EASTWOOD, Naut. Alm. Office, Cambridge, Mass.
Prof. C. W. ELIOT, Harvard Coll., Cambridge, Mass.
*Mr. E. B. ELLIOTT, Consulting Actuary, Boston.
*Prof. FERDINAND ENGEL, U. S. Coast Survey, Washington, D. C.
*Prof. H. L. EUSTIS, Lawrence Scientific School, Cambridge, Mass.
Prof. E. W. EVANS, Marietta Coll., Ohio.
Hon. EDWARD EVERETT, Boston.
Prof. BENJAMIN S. EWELL, William & Mary's Coll., Williamsburg, Va.
Prof. J. H. FAIRCHILD, Oberlin Coll., Ohio.
*Prof. W. FERREL, Nashville, Tenn.
Prof. M. H. FISK, Paducah Coll., Kentucky.
Mr. OSCAR C. FOX, Prin. Nelson Academy, Ohio
Mr. JAMES B. FRANCIS, Civil Engineer, Lowell, Mass.
Prof. JOHN F. FRAZER, Univ. of Penn., Philadelphia.
Prof. JOHN R. FRENCH, Prin. Mexico Academy, N. Y.
*Prof. EDWARD T. FRISTOE, Columbian Coll., Washington, D. C.
Prof. H. R. GEIGER, Wittenburg Coll., Springfield, Ohio.
Prof. W. M. GILLESPIE, Union Coll., Schenectady, N. Y.
*Dr. B. A. GOULD, Jr., Director Dudley Obs., Albany, N. Y.
Prof. DASCOM GREENE, Rensselaer Institute, Troy, N. Y.
*Prof. CHARLES W. HACKLEY, Columbia Coll., N. Y.
*Mr. D. B. HAGAR, Prin. Eliot High School, Jamaica Plain, Mass.
*Prof. R. D. HAMILTON, Hobart Free Coll., Geneva, N. Y.
Prof. B. S. HEAD, Missouri Univ., Columbia
*Mr. J. B. HENCK, Civil Engineer, Boston.
*Prof. W. D. HENKLE, Richmond, Indiana.
*Prof. JOSEPH HENRY, Sec'y Smithsonian Inst., Washington, D. C.
Mr. EDWARD C. HERRICK, Librarian Yale Coll., New Haven, Conn.
Prof. ROBERT A. HIGGINS, Semple-Broaddus Coll., Centre Hill, Miss.
*Mr. J. E. HILGARD, U. S. Coast Survey, Washington, D. C.
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Prof. ALONZO JACKMAN, Norwich Univ., Norwich, Vt.
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*Mr. J. F. KELLOGG, Providence Conference Seminary, East Greenwich, R. I.

*Prof. E. Otis Kendall, Univ. of Penn., Philadelphia.
*Prof. W. C. Kerr, Davidson Coll., N. C.
Prof. Daniel Kirkwood, Indiana Univ., Bloomington.
Prof. Benjamin L. Lang, Kenyon Coll., Gambier, Ohio.
Prof. Harvey B. Lane, Wesleyan University, Middletown, Conn.
Prof. N. R. Leonard, Yellow Springs Coll., Iowa.
*Prof. Edmund Longley, Emory & Henry Coll., Virginia.
Mr. E. J. Loomis, Naut. Alm. Office, Cambridge, Mass.
Prof. Elias Loomis, New York University, N. Y.
Prof. Joseph Lovering, Harvard Coll., Cambridge, Mass.
Prof. M. Lyford, Waterville Coll., Maine.
Prof. D. H. Mahan, Military Academy, West Point.
*Mr. James Main, U. S. Coast Survey, Washington, D. C.
Prof. J. P. Marshall, Tufts Coll., Medford, Mass.
Rev. Dr. McCawley, President King's Coll., Windsor, Nova Scotia.
Mr. Abijah McLean, New Lisbon, Columbia Co., Ohio.
*Mr. L. W. Meech, U. S. Coast Survey, Preston, Conn.
Prof. J. L. Meigs, LaGrange Synodical Coll., Tenn.
President Robert Milligan, Kentucky University, Harrodsburg.
Prof. O. M. Mitchell, Director Obs., Cincinnati, Ohio.
Prof. R. M. Moore, M'Kendree Coll., Lebanon, Ill.
Mr. E. G. Morrow, Naut. Alm. Office, Cambridge, Mass.
Prof. A. L. Nelson, Washington Coll., Lexington, Va.
President C. K. Nelson, St. John's Coll., Annapolis, Md.
*Mr. Simon Newcomb, Naut. Alm. Office, Cambridge, Mass.
*Prof. W. W. Newman, Buffalo, New York.
*Prof. H. A. Newton, Yale Coll., New Haven, Conn.
Mr. W. L. Nicholson, U. S. Coast Survey, Washington, D. C.
*Mr. James E. Oliver, Naut. Alm. Office, Lynn, Mass.
Prof. Denison Olmsted, Yale Coll., New Haven, Conn.
Prof. L. M. Osborn, Madison University, Hamilton, N. Y.
Prof. W. H. Parker, Middlebury Coll., Vermont.
Mr. John Paterson, Albany, N. Y.
Prof. J. W. Patterson, Dartmouth Coll., Hanover, N. H.
*Prof. Benjamin Peirce, Harvard Coll., Cambridge, Mass.
Mr. James Mills Peirce, Tutor in Harvard Coll. Cambridge, Mass.
*Prof. George R. Perkins, Utica, N. Y.
Dr. C. H. F. Peters, Director of Obs., Hamilton Coll., Clinton, N. Y.
Prof. McKendree Petty, Univ. of Vermont, Burlington.
Prof. Wm. F. Phelps, Prin. Normal School, Trenton, N. J.
*Prof. Charles Phillips, Univ. of North Carolina, Chapel Hill.
Prof. James Phillips, Univ. of North Carolina, Chapel Hill.
Mr. Pike Powers, Staunton, Va.
Prof. J. M. Pugh, Madison Coll., Sharon Miss.
Hon. S. S. Randall, Supt. Public Instruction, New York City.
Prof. J. J. Reynolds, Washington Univ., St. Louis, Mo.
President Werden Reynolds, Worcester Female College, Worcester, Mass.
Mr. H. W. Richardson, Tutor in Waterville Coll., Maine.
*Prof. J. M. Richardson, Bowdon, Georgia.
Prof. H. N. Robinson, Elbridge, Onondaga Co., N. Y.
Prof. Fairman Rogers, Univ. of Penn., Philadelphia.
Prof. Wm. B. Rogers, Boston.
*Prof. O. Root, Hamilton Coll., Clinton, N. Y.
Prof. J. J. Rucker, Georgetown Coll., Kentucky.
Prof. J. W. Rust, Bethel Female Coll., Hopkinsville, Ky.
Prof. Alfred Ryors, Centre Coll., Danville, Kentucky.
*Mr. Truman Henry Safford, Naut. Alm. Office, Cambridge, Mass.
Mr. A. Schuyler, Republic, Seneca Co., Ohio.
Prof. Benedict Sestini, Georgetown Coll., D. C.
*Mr. Thomas Sherwin, Prin. English High School, Boston.
Dr. A. W. Smith, Middletown, Conn.

Col. Francis H. Smith, Supt. Virginia Military Institute. Lexington.
Prof. F. H. Smith, Univ. of Virginia, Charlottsville.
Prof. E. S. Snell, Amherst Coll., Amherst, Mass.
*Mr. Charles H. Sprague, Naut. Alm. Office, Malden, Mass.
*Prof. M. C. Stevens, Haverford Coll., West Haverford, Pennsylvania.
President W. M. Stewart, Stewart Coll., Clarksville, Tenn.
Prof. J. F. Stoddard, Montrose, Susquehanna Co., Penn.
Prof. T. Strong, Rutgers Coll., New Brunswick, N. J.
Prof. John Tatlock, Williams Coll., Williamstown, Mass.
Prof. John Towler, Hobart Free Coll., Geneva, N. Y.
*Prof. W. P. Trowbridge, U. S. Coast Survey, Washington, D. C.
*Prof. J. M. Van Vleck, Middletown, Conn.
*Mr. G. B. Vose, U. S. Coast Survey, Washington, D. C.
Prof. W. T. Walters, Wake Forest Coll., Forestville, N. C.
Mr. Wm. R. Ware, Architect, Boston.
*Mr. William Watson, Instructor, Lawrence Scientific School.
*President N. B. Webster, Va. Collegiate Institute, Portsmouth, Virginia.
Prof. Henry H. White, Kentucky Univ., Harrodsburg.
*Prof. Geo. C. Whitlock, Victoria Coll., Coburg, Canada West.
Lieut. Wm. H. Willcox, U. S. N., Naval Academy, Annapolis, Md.
Prof. J. C. Wills, Randolph-Macon Coll., Virginia.
Prof. William Wines, Stewart Coll., Clarksville, Tenn.
*Prof. Joseph Winlock, Sup't Am. Naut. Alm., Cambridge, Mass.
Prof. Devolson Wood, Univ. of Michigan, Ann Arbor.
Prof. J. S. Woodman, Dartmouth Coll., Hanover, N. H.
*Mr. Chauncey Wright, Naut. Alm. Office, Cambridge, Mass.
Prof. Charles A. Young, Western Reserve Coll., Hudson, Ohio.
*Prof. W. H. Young, Ohio Univ., Athens.
Prof. Ira Young, Dartmouth Coll., Hanover, N. H.

ADDITIONAL NAMES.

Prof. Ira W. Allen, Antioch Coll., Yellow Springs, Ohio.
President John Barker, Allegheny Coll., Meadville, Penn.
*Mr. W. P. G. Bartlett, Naut. Alm. Office, Cambridge, Mass.
Mr. Samuel P. Bates, Fountain Side, Meadville, Penn.
Mr. George N. Bigelow, Prin. Normal School, Framingham, Mass.
Mr. James G. Board, Hendrick's Store, Bradford Co., Va.
Prof. W. R. Boyers, Principal Buffalo Academy and Seminary, Buffalo, Putnam Co., Va.
Mr. E. E. Bradbury, Prin. High School, Ware, Mass.
Prof. John B. Cary, Principal Hampton Academy, Hampton, Va.
Prof. C. W. H. Cathcort, Dayton, Ohio.
Mr. Pliny Earle Chase, Philadelphia, Penn.
Mr. Marshall Conant, Prin. Normal School, Bridgewater, Mass.
Prof. Dorsey Cox, Burlington Coll., N. J.
Prof. M. A. Cummings, New Hampton Literary and Theological Institution, Fairfax, Vt.
Prof. Sylvester Dixon, N. H. Conference Seminary, Sanbornton Bridge, N. H.
Mr. William Edson, Civil Engineer, Boston, Mass.
Prof. Richard Ellis, Hudson River Institute, Claverack, N. Y.
Mr. Joseph Ficklin, Jr., Prin. High School, Trenton, Mo.
Mr. J. Foster Flagg, Civil Engineer, Washington, D. C.
Prof. Lewis R. Gibbes, Coll. of Charleston, S. C.
Prof. B. Franklin Greene, Director Rensselaer Polytechnic Institute, Troy, N. Y.
Mr. S. P. Hale, Prin. High School, Mt. Harmony, Tenn.
*Prof. Benjamin Hallowell, Alexandria, Va.
Mr. Sylvanus Hayward, Pembroke, N. H.

Prof. B. S. Hedrick, New York City.
*Dr. Joel E. Hendricks, Newville, Indiana.
Prof. Frederick Humphrey, State Univ., Iowa City.
Mr. William H. Johnson, Supt. Schools, Buckingham, Bucks Co., Penn.
Prof. C. H. Judson, Furman Univ., Greenville, S. C.
Mr. Ernst Krauss, North Granville, N. Y.
Prof. C. W. Lane, Oglethorpe Univ., Talmage, Ga.
Mr. J. Homer Lane, Washington, D. C.
Prof. James H. Magoffin, Stewart Coll., Clarksville, Tenn.
Prof. Alfred M. Mayer, Baltimore Coll., Baltimore, Md.
Mr. James McClung, Tipton, Cedar Co., Iowa.
Prof. R. W. McFarland, Miami Univ., Oxford, Ohio.
Prof. Henry Miles, Univ. of Bishop's Coll., Lennoxville, Canada East.
Col. E. W. Morgan, Supt. Military Institute, near Frankfort, Ky.
Mr. F. E. Page, F. B. School, Providence, R. I.
Prof. J. C. Porter, Pittsburg, Pa.
Prof. E. T. Quimby, Prin. Appleton Academy, New Ipswich, N. H.
Mr. Smith Ragsdale, Clarksville, Texas.
Hon. Andrew J. Rickoff, Supt. Schools, Cincinnati, Ohio.
Prof. J. L. Riddell, Univ. of Louisiana, N. O.
Mr. Ira Sayles, School Commissioner, 1st Dist., Alleghany Co., Rushford, Penn.
Lieut. J. M. Schofield, Military Academy, West Point.
Thomas W. Silloway, Esq., Architect of the new Capitol, at Montpelier, Vt.
Prof. William Smyth, Bowdoin Coll., Brunswick, Maine.
Mr. Zelotes Truesdel, Principal Union High School, Moline, Ill.
Prof. W. J. Vaughn, Univ. of Alabama, Tuscaloosa.
Hon. W. H. Wells, Supt. Schools, Chicago, Ill.
Prof. D. W. C. Williams, Miss. Coll., Clinton, Miss.
Mr. Solomon Wright, Lahaska, Bucks' Co., Penn.
Prof. Jeffries Wyman, Harvard Coll., Cambridge, Mass.

We had intended to give such extracts from these replies as would indicate their nature; but are reluctantly compelled, from their number and length, to substitute the following summary: —

To the question, "Do you think there is a present need of a mathematical journal of any kind?" the replies are unanimous in the affirmative. To the question, "Do your views coincide with those here expressed as to its character?" the replies are unanimous in the affirmative. To the question, "Are you willing to assist in establishing and sustaining a journal, by contributing to its pages?" the replies, with very few exceptions, promise such aid as may be consistent with other duties; while those whose names are marked with a star have pledged constant and active coöperation.

The following Report and Resolution were unanimously adopted in the Section of Mathematics and Physics, at the meeting of the American Association for the Advancement of Science, held at Baltimore.

"The Committee to whom was referred Mr. Runkle's project for the establishment of a Mathematical Journal, have examined the subject, and submit their Report as comprised in the following Resolution, which they recommend to the adoption of the Section.

"*Resolved*, That we deem the establishment of a Mathematical Journal upon the plan proposed by Mr. J. D. Runkle, if well executed, to be highly important to the mathematical progress of the country, and the advancement of science; and that we have full confidence in Mr. Runkle to do justice to the task which he has undertaken.

A. Caswell,
Benjamin Peirce,
Geo. W. Coaklay."

"*April* 30, 1858."

At the late meeting of the New Hampshire State Teachers' Association, held at Concord, the following Resolution, introduced by Prof. J. W. Patterson, of Dartmouth College, was unanimously adopted.

"*Resolved*, That we feel a deep interest in the establishment of a Mathematical Journal of such a character as that proposed by Mr. J. D. Runkle, of Cambridge, and that we will do all in our power to favor and sustain its publication, believing as we do, that it will tend greatly to promote the progress of pure and mixed mathematics, and to the advancement of the science."

At the recent meeting of the Iowa State Teachers' Association, the following Resolution was adopted.

"*Resolved*, That this Association cordially indorses the project of establishing a Mathematical Journal on the plan proposed by Mr. J. D. Runkle, of Cambridge, and commends it to the favorable consideration and support of the Teachers of Iowa.

G. W. Hough,
F. Humphrey, } *Committee*."
J. McClung,

RUNKLE'S PRIZES

TO THE STUDENTS IN ANY INSTITUTION OF LEARNING IN THE UNITED STATES OR BRITISH PROVINCES.

I. For Solutions.

Judges.

Prof. Joseph Winlock, Mr. Chauncey Wright,
Mr. Truman Henry Safford.

In the first and each succeeding number of the Journal during the year, we shall publish five problems, entitled *Prize Problems for Students.* The student who shall send us the best solutions of the greatest number of the prize problems, in any number of the Journal, in time for the third number following the one in which they are proposed, shall be entitled to a first prize of *ten dollars;* the second in order of merit shall be entitled to a second prize of *a bound copy of the first volume of the Journal.* Each solution will be estimated by the judges independently, and to the students who have the first and second highest aggregates of marks shall be awarded the prizes. The best solution of each problem will be published, entitled *Prize Solution,* with such other solutions as the judges shall consider of sufficient merit. All the steps in each solution must be fully given, and the whole communicated in a plain and legible handwriting, to secure attention. The award of the judges, announcing the names of the successful competitors, the prize solutions with the names of the authors, and credit for all solutions received, will be published in the fourth and succeeding numbers. No student shall be entitled to the same prize twice during the same year; but full credit will be given him in the award of the judges.

II. For Essays.

Judges.

Prof. W. Ferrel, Mr. J. B. Henck,
Prof. George R. Perkins.

A first prize of *fifty dollars* for the best essay.

A second prize of *forty dollars* for the next in order of merit.

A third prize of *thirty dollars* for the third in order.

A fourth prize of *twenty dollars* for the fourth.

A fifth prize of *ten dollars* for the fifth.

These prizes will be given for essays upon any questions in pure or applied mathematics, including those questions in physics "which can be solved only by an application of mathematical logic to the fundamental principles which constitute the scientific conception of the phenomena."

The essays are not to be simply descriptive, but thoroughly digested discussions of the questions under consideration, presented with clearness and symmetry. Originality in matter or treatment, or in both, is certainly desirable; but even without these, an essay may show such a complete mastery of the subject as to be highly meritorious.

Essays must be brief, not exceeding in length eight pages of the Journal, and must be judged worthy of publication to be entitled to any prize.

They must be received at or before the date of publication of the eighth number, that the award of the judges may be announced, and the essays published in the first volume.

THE

MATHEMATICAL MONTHLY.

Vol. I....OCTOBER, 1858....No. I.

POSTSCRIPT TO INTRODUCTORY NOTE.

We have included the foregoing pages in our first Number to save any repetition that might otherwise be necessary for those who have not had their attention called to the subject; and also to put in more permanent form all that relates to the history of the enterprise. The whole subject of the establishment of the Mathematical Monthly, including its proposed character, its aims and ends, and the proper means to accomplish them, and its educational as well as scientific bearing has been so carefully considered by so large a number of those whose opinions are entitled to the greatest respect, that nothing more remains for us, in this connection, than to add, that we shall endeavor to carry out the enterprise in the spirit of its conception, and to exercise our best judgment in the use of the material placed at our disposal. We may not, however, omit this opportunity to express our sincere thanks to the many in all parts of the country, to the press as well as to individuals, for the active interest taken in the enterprise, and to acknowledge that to this influence our present favorable prospects are mainly due. We wish to be considered responsible only for such communications as appear anonymously in the Monthly.

PRIZE PROBLEMS FOR STUDENTS.

I.

FIND θ from each of the equations,—

$$\tan\theta\tan 2\theta + \cot\theta = 2,$$
$$2\sin^2 3\theta + \sin^2 6\theta = 2,$$
$$\cos n\theta + \cos(n-2)\theta = \cos\theta.$$

II.

The whole surface of a cone is three times the area of the base. Find its vertical angle.

III.

The sum of the squares of the reciprocals of two radii vectores from the centre of an ellipse, at right angles to each other, is constant; the perpendicular from the centre, on the chord joining their extremities, is also constant. What part of the area of the ellipse is the circle whose radius is this perpendicular?

IV.

Two circles whose radii are R and r touch each other externally. If θ is the angle included between the common tangents to the two circles, prove that

$$\sin\theta = \frac{4(R-r)\sqrt{Rr}}{(R+r)^2}.$$

V.

Four circles may be described, each of which shall touch the three sides of a plane triangle or those sides produced. If six straight lines be drawn joining the centres of these circles, two and two, prove that the middle points of these six lines are in the circumference of a circle circumscribing the given triangle.

The solutions of these problems must be received by the 10th of December, in order that the awards of the judges may be announced in the number for January, 1859. See page xi. of the Introduction.

Competitors will send their names, with the names and localities of the institutions with which they are connected, on a separate slip, to be retained by the editor until the awards are made. These slips must also be signed by their Instructors, as evidence that the parties are fairly entitled to compete for the prizes.

Essays must be received by May 1st, 1859, and competitors for these prizes will comply with the directions just specified for problems. Also see page xii. of the Introduction.

We respectfully invite the attention of teachers of mathematics to the subject, and ask them to coöperate with us in giving these prizes a fair trial. If the good resulting shall be found at all in proportion to the outlay involved, some means will be found to continue them beyond our first year.

NOTE ON DECIMAL FRACTIONS.

BY REV. THOMAS HILL.

WE are convinced, both from theoretical considerations, and from experiment in the school-room, that a child should learn the use of decimal fractions as soon as he does that of whole numbers. In order that he may do this, the analogy between decimal and vulgar fractions must, at first, be kept sedulously out of sight; and the decimal fraction be treated as a natural and necessary extension of the decimal law of notation, to the right hand of the unit's place. Unless the child is thus early familiarized with the use of decimals, they never become, as they should to all arithmeticians, the most natural and easy mode of treating quantities, not simple multiples of the unit.

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RULE FOR FINDING THE GREATEST COMMON DIVISOR.

BY PLINY EARLE CHASE.

When the greatest common measure of more than two quantities is required, a rule like the following is usually given: —

"First, find the greatest common measure of any two of the numbers, then find the greatest common measure of the number found and another of the given numbers, and thus proceed till all the given numbers are brought in."

There is usually one or more of the given numbers that can be readily resolved by inspection into submultiples, and the divisor sought can thus be found very readily. But if the prime factors of the numbers are all large, the following rule is more expeditious than any other one that I have ever seen.

1. Divide all the given numbers by the least of them, and bring down the remainders.

2. Divide the first divisor and all of the first remainders by the least of them, and bring down the remainders.

3. Proceed in this manner until a remainder is found that will measure all the other remainders, and the divisor last used, — and this will be the greatest common divisor.

Every abbreviation that can be used at any step of the process, such as rejecting factors that are evidently not common, &c., should of course be employed.

EXAMPLES.

1. Find the greatest common measure of 940, 747, 529, and 551. By inspection we resolve 940 into the factors $2^2 . 5 . 47$; $2^2 . 5 .$ being evidently not common, are rejected, and we ascertain by a single trial that 47 is not a common factor. The answer is therefore 1.

2. Required the greatest common divisor of 1633, 3763, 4757, and 4189. Proceed, according to our rule, as follows: —

1633)3763	4757	4189
3266	3266	3266
497	1491	923
497)1633	1491	923
1491	1491	497
$142 = 2 \times 71$		$426 = 6 \times 71$
71)497		
497		*Ans.* 71.

NOTE ON EQUATION OF PAYMENTS.

BY G. P. BOND.

THE time at which two or more accounts, bearing interest from different dates, may be settled by a single payment of a sum equal to the total amount of all the debts, is found, according to the rule commonly used, in the following way.

Multiply each debt by the time that must intervene before it becomes due, and divide the sum of the products by the sum of the debts. The quotient will be the interval of time required.

If we wish to find the distance of the centre of gravity of a number of weights suspended on a straight rod, measured from a given point in the rod, we *multiply each weight by its distance from this point, and divide the sum of the products by the sum of the weights. The quotient will be the distance required.*

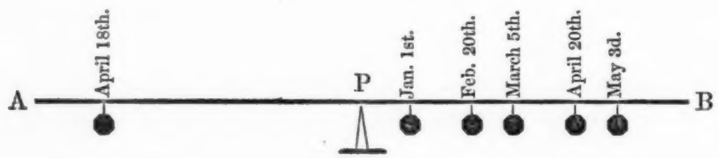
The analogy between the two processes suggests an easy mechanical method of computing the equation of payments, which we will illustrate by an example.

A merchant owes the following sums, and requires to know the

time at which, by a single payment equal to the sum of the several debts, all the accounts will be settled with interest.

	Debts.	Bearing Interest from
	$500	Jan. 1,
	260	Feb. 20,
	110	March 5,
	50	April 20,
	5000	May 3.
Total,	5920	

In the annexed figure, A B is a bar of wood or metal balanced at P, and graduated with equal divisions to the months and days of one or more years, on each side of P.



At the graduation corresponding to Jan. 1st is hung a weight representing $500, at the date Feb. 20th another of $260, and so on, representing each sum by a proportional weight suspended from the bar at the proper dates. On the opposite side of P is hung a weight equal to the sum of all the other weights. The date (in this example, April 18th), at which it must be placed in order to restore the balance of the bar, is the time when the payment of the total sum of $5920 will discharge all the debts with interest.

The chief difficulty with the apparatus is to apportion the weights, but no great nicety will be needed, especially as fractional parts of a day and the difference between discount and interest paid in advance, are commonly disregarded in such settlements.

ON THE STUDY OF GEOMETRY.

BY SAMUEL P. BATES.

We propose in this paper to throw out a hint concerning the study of Geometry, which has escaped the notice of authors on this science, though it may have received the attention of teachers. Our purpose is to present matter which is practical, rather than that which is profound.

The object of a liberal course of study in the mathematics is to develop the faculties of abstraction, reason, and imagination. Few, of the many who pursue these studies, ever make any practical use of them, beyond the discipline and strength thereby acquired. It should therefore be the aim of those who direct the studies of pupils, to arrange the course of instruction so as to secure the most complete development.

Most of those who study geometry as it is taught in our seminaries and colleges, simply aim at acquiring a just conception of the several steps in the demonstrations, as they are set down in the text-book. This course, if thoroughly pursued, will vastly strengthen the reasoning powers. But from this alone does not result that high order of discipline, which ought to be gained by the study of this noble science.

The pupil should, in addition, be required to construct the series of syllogisms necessary to establish the demonstration, from that which enunciates the theorem, back to the axioms or demonstrated propositions on which the proof rests. This would serve to quicken the powers of analysis and generalization, in which consists richness of thought. The bearing of each step in the demonstration would thus be clearly discovered, and the beauty of the proof would be apparent.

But learning the demonstration of a proposition as it is given in the book, or even constructing the syllogisms which are implied in the proof, will not improve the imagination. Hence the pupil should not only be required to learn the demonstrations given, but to construct them for himself. A text-book properly prepared should furnish the demonstrations of only a portion of the propositions; the remainder should be enunciated, and the pupil required to exercise his ingenuity in discovering the proof of them.

I will illustrate my idea with an example. Suppose the text-book to furnish the demonstration of the celebrated forty-seventh proposition of Euclid, and following it the enunciation of this

Theorem. The triangles, formed by joining the exterior angles of the squares erected on the three sides of a right-angled triangle, are equivalent.

It is asserted that the three triangles thus formed are equivalent; it is required to prove it. The imagination is now set to work. The inquiry is at once suggested, May not the three triangles thus formed be each equivalent to the original triangle? If they are, and we can succeed in proving it, our object is attained. Some such inquiry as this must first be started. The imagination reaches out for an hypothesis; and this process must be continued until we have a theory which we can prove. We may discover the correct theory, and still not be able to construct the demonstration; but we can never construct the demonstration till we have the theory. Franklin, in his discoveries in electricity, was obliged to start with the inquiry, Is not the electrical spark identical with the flashes of lightning? This was his hypothesis, and, after repeated experiments, he succeeded in proving it true.

We thus perceive that the same faculties are required for the discovery of a geometrical demonstration as are employed in making discoveries in any of the sciences. If pupils were required to con-

struct demonstrations for themselves, while pursuing the study of geometry, their powers of original investigation would thereby be greatly improved. We sincerely hope that teachers may not only talk about this subject in their lecture rooms, but may make a practical application of it in their classes.

DEMONSTRATION OF A THEOREM.

BY PIKE POWERS.

"*Two regular polygons, of the same number of sides, can always be formed, the one circumscribed about a circle, the other inscribed in it, which shall differ from each other by less than any assignable quantity.*"

Let $A =$ area of any inscribed regular polygon,
$B =$ area of a similar circumscribed polygon,
$A' =$ area of inscribed polygon of twice as many sides,
$B' =$ area of circumscribed polygon of twice as many sides.

Then $\quad A' = \sqrt{A.B}, \; B' = \dfrac{2\,A.B}{A+A'}$, (see Davies' Legendre).

Hence $\quad B' - A' = \dfrac{2\,A.B}{A+A'} - A' = \dfrac{2\,A\,B - A\,A' - A'^2}{A+A'}$

$$= \frac{A\,B - A\,A'}{A+A'} = \frac{A}{A+A'}(B - A').$$

Now $\quad A' > A \therefore B - A' < B - A, \dfrac{A}{A+A'} < \dfrac{A}{2\,A} < \frac{1}{2}$,

and $\quad \dfrac{A}{A+A'}(B - A') < \frac{1}{2}(B - A)$.

Hence, $\quad B' - A' < \frac{1}{2}(B - A)$.

In like manner $B'' - A'' < \frac{1}{2}(B' - A') < \frac{1}{2^2}(B - A)$,

and so on to $\quad B^n - A^n < \frac{1}{2^n}(B - A)$,

where $B^n - A^n$, the difference between the inscribed and circumscribed polygons, is infinitely small, when n is infinitely great.

DEMONSTRATION OF THE PYTHAGOREAN PROPOSITION.

BY JAMES EDWARD OLIVER.

The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

Drop a perpendicular from the right angle to the hypothenuse, and prove in the usual way that the two partial triangles thus formed are similar to the original one. On the hypothenuses of these three triangles, that is, on the three sides of the original triangle, as homologous sides, describe three figures similar to each other (squares for instance). Their areas are evidently equimultiples of the respective triangles; but two of the triangles are seen to make up the third; hence, the two corresponding squares will together make up the square on the hypothenuse. *Q. E. D.*

Perhaps the property, that similar areas are to each other as the squares of their homologous sides, being much simpler than the Pythagorean Proposition, and not depending upon it, should naturally precede it.

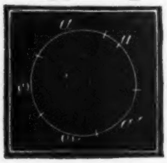
NOTE ON NAPIER'S RULES.

BY TRUMAN HENRY SAFFORD.

In the form in which they are usually given, the rules are —

I. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

II. *The sine of the middle part is equal to the product of the cosines of the opposite parts.*



Rule I., as will be shown, is superfluous. Represent the parts in the following order, as it were around a circle, and if μ be the middle part, α and α' the adjacent parts, ω and ω' the opposite parts, we shall have,

by Rule II., the equation

$$\sin \mu = \cos \omega' \cos \omega . \tag{1}$$

But if ω' be the middle part, ω and α' will be adjacent parts, μ and α opposite parts, and by Rule II.,

$$\cos \mu \cos \alpha = \sin \omega'. \tag{2}$$

If we add together the squares of (1) and (2), we shall have

$$\sin^2 \mu + \cos^2 \mu \cos^2 \alpha = \cos^2 \omega \cos^2 \omega' + \sin^2 \omega' ;$$

or

$$1 - \cos^2 \mu \sin^2 \alpha = 1 - \cos^2 \omega \sin^2 \omega' ;$$

or

$$\cos^2 \mu \sin^2 \alpha = \cos^2 \omega' \sin^2 \omega ;$$

or

$$\cos \mu \sin \alpha = \pm \cos \omega' \sin \omega. \tag{3}$$

Equations (1), (2), and (3) together are sufficient for the determination of any two parts, two others being known; provided the sign of the second term of (3) is known. In the case of ordinary spherical triangles the sign is positive.

Dividing (3) without its sign by (2), we shall have

$$\tan \alpha = \cot \omega' \sin \omega ;$$

or

$$\sin \omega = \tan \alpha \tan \omega'. \tag{4}$$

If ω be the middle part, α and ω' will be adjacent parts, so that Rule I. is the same as Equation (4), a deduction from Rule II. alone.

In Napier's rules the parts are, the two legs, the complements of the hypothenuse and two of the angles; the right angle not being counted.

PROBLEM, BY PROF. PEIRCE.

WHAT curve is represented by the equation

$$\sin \varepsilon = a \, {}^*\mathrm{Cos} \, (\log r + b),$$

in which r denotes the radius vector, and ε the angle between r and the corresponding tangent.

* Whenever in trigonometric notation, we shall use initial capital letters, potential functions will be indicated.

ON THE RELATION BETWEEN THE STATES OF MINIMUM AND EQUILIBRIUM.

BY JOHN PATTERSON.

1. THE phenomenon of minimum is one of stable equilibrium, that of maximum being in general unstable. In the same plane, the shortest route from the point A (fig. 1) to the point B, by the way of the line $L L'$, is well known to be through the point P on the given line, where the angles $A P M$ and $B P M$ of the two branches of the path with the normal $P M$, as also the angles $A P L$ and $B P L'$ of the same branches with the line $L L'$, are respectively equal.

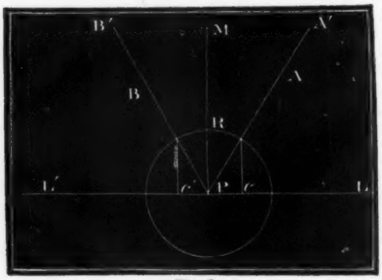


Fig. 1.

This is the familiar example of the reflection of light from a plane surface. Pass now to the dynamical form of the phenomenon. The line $L L'$ is rigid, and fixed in direction; the point P is acted upon by two equal forces, in the respective directions $A'P$ and $B'P$; and as it is immaterial at what particular point in either of these rigid lines AP, $B P$ the respective forces originate or are applied, so long as they are equal, and act uniformly in the directions stated, their effects upon the point P will be equal, and the point cannot slide upon the line, because the components of the forces in its direction are equal in magnitude and opposite in direction. With any radius $P R$ describe a circle on P; and the deductions from the example are,

1°. At the sought point P, the normal to the given line $L L'$ must make equal angles with the directions of the given origins of the equal forces; and

2°. The trigonometrical cosines $P c$ and $P c'$, or components of the unit measure of the respective forces on the line $L L'$, must be equal and opposed.

Then if the point P be given instead of the line $L L'$, the direction of this equilibrating line will be found from the normal $P M$ obtained by dividing equally the angle $A P B$.

2. Instead of two forces encountering a fixed line, the point P may be held in equilibrium by any number n of equal forces acting equally upon it, provided they form, two and two, equal angles about that point; for then the algebraical sum of the cosines, estimated in any direction through the point P, will be zero, as may be seen in the figure drawn for $n = 5$. Therefore when the points of application of five equal forces are given, the construction indicated for the point of equilibrium consists in describing on each chord $A B$, $B C$, $C D$, $D E$, $E A$ (fig. 2), an arc of circle to contain an angle of 72°; they will all intersect in the point O of equilibrium, or, indeed, the intersection of any two of the circles is sufficient.

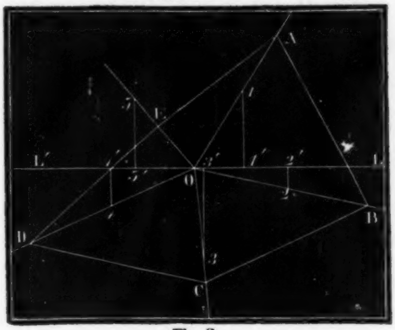


Fig. 2.

To show that O, the point of equilibrium, is also the point of aggregate minimum distance from the vertices of the irregular pentagon $A B C D E$ (fig. 3), draw perpendiculars $Q a$, $Q b$, $Q c$, $Q d$, $Q e$ from any other point Q upon $A O$, $B O$, $C O$, $D O$, $E O$: then it is easily proved that the sum of the prolongations $O c + O d + O e$ is equal to the sum of the curtail-

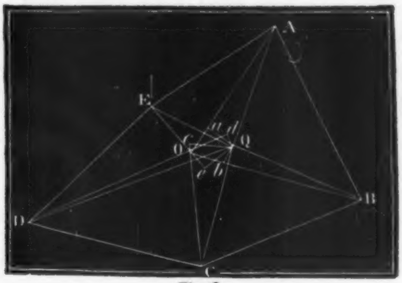


Fig. 3.

ments $Oa + Ob$ of the lines passing from the vertices through the point O; whence

$$Aa + Bb + Cc + Dd + Ee = AO + BO + CO + DO + EO.$$

Now draw AQ, BQ, CQ, DQ, EQ; and each of these lines is the hypothenuse of a right angled triangle AaQ, BbQ, CcQ, DdQ, EeQ, and therefore greater than its corresponding base Aa, Bb, Cc, Dd, Ee. Consequently

$$AQ + BQ + CQ + DQ + EQ > AO + BO + CO + DO + EO.$$

3. The state of minimum consists, therefore, of the equilibrium of equal translatory forces upon a point: if the forces come to vary unequally, or to be arbitrarily changed in magnitude, the position of the point of equilibrium will change, and, although still remaining the equilibrating centre of the system it will no longer be the point of minimum aggregate distance simply, but the minimum of distances under the changed operation of the forces.

4. When $n = 3$, the problem requires the determination of the point of minimum aggregate distances from the three vertices A, B, C of a triangle with given sides a, b, c. This problem was solved by Viviani, and published by him in his Latin treatise on *Geometrical Divinations*. The question was communicated to Viviani by his friend and fellow-student, Torricelli, to whom it appears to have been proposed by Fermat as a sort of challenge. Viviani confesses in a note that the question had cost him much effort. He proved geometrically that the three angular points of the triangle must subtend equal angles at the minimum point. It is also asserted that Torricelli himself effected the solution by three different methods, and each differing from that of Viviani. Subsequently the question was discussed by many different hands, and under various aspects; and a complete general solution is given in Simpson's *Treatise on Fluxions*, applicable to any number of points, and to different relations of the forces applied at them. (End in No. II.).

NOTE ON DERIVATIVES.

BY REV. THOMAS HILL.

THERE are certain propositions in algebra which may be more readily demonstrated by the aid of derivatives than in any other way. As brevity and clearness of reasoning are prime requisites for a successful mathematician, I should advocate the introduction of the doctrine of derivatives at a very early stage in algebraical studies. The following form will be found, I think, sufficiently simple for young students, and sufficiently comprehensive to be used to great advantage in abbreviating modes of demonstration.

1. As an unknown quantity, x, may be conceived of as of any value, so it may also be conceived as increasing or diminishing so as to pass successively through various values. A quantity thus conceived of as changing its values, is called a variable.

2. Every algebraic expression containing x is called a function of x.

3. If a variable, x, be multiplied by a constant, a, the product, $a x$, will manifestly change a times as fast as x changes. The ratio between the rate of change in a function and that in the variable, is called the derivative of the function. Thus a is the derivative of the product $a x$. But if a were the variable, the derivative relatively to a would be x; that is $a x$ would change x times as fast as a changed.

4. If x were a quantity dependent on y (for instance if $x = 2y$), the derivative of $a x$ would be a times the derivative of x, taken relatively to y; (that is, in the supposed case, a times 2, or $2a$).

5. If the variable be taken n times as a factor, it is manifest that the rate of change in each factor, x, would be multiplied by all the other $n - 1$ factors, each of which is x, so that the rate of change in each factor would be multiplied by x^{n-1}. But as there are n

factors, the whole rate of change in x^n is $n x^{n-1}$ as fast as that in x. In other words the derivative of x^n is $n x^{n-1}$, and of $a x^n$, it is $n a x^{n-1}$.

6. The same reasoning as that in § 5 would show, that in the nth power of any function of x, the rate of change in each function is multiplied by the other functions, and the total derivative will be the derivative of the function multiplied by n times the function raised to the $(n-1)$st power.

7. D_x signifies derivative relatively to x. Thus

$$D_x x = 1. \quad D_x a x = a. \quad D_x a x^n = n a x^{n-1}.$$

$$D_x (a + 2x)^3 = 3(a + 2x)^2 D_x (a + 2x) = 2.3(a + 2x)^2.$$

$$D_x (a + 7bx^2)^5 = 5(a + 7bx^2)^4 D_x (a + 7bx^2) = 70bx(a + 7bx^2)^4.$$

8. A derivative of a derivative is called the second derivative of the function, and these higher derivatives are denoted by exponents like those of powers.

$$D_x a x^n = n a x^{n-1},$$

$$D_x^2 a x^n = n(n-1) a x^{n-2},$$

$$D_x^3 a x^n = n(n-1)(n-2) a x^{n-3}.$$

Applications to the development of the Binomial, Taylor, and McLaurin's Theorems in the next number.

PROPOSITIONS ON THE DISTRIBUTION OF POINTS ON A LINE.

BY PROF. BENJAMIN PEIRCE.

The following propositions are given without demonstration, in the hope that they will incite some one to a thorough investigation

of the subject, or that at least the various forms of demonstration, which will undoubtedly suggest themselves to different minds, may, if they are sent to the Journal, furnish the elements of further inquiry.

1. If two points are taken in all possible relative positions upon a given line, their distance apart in one half of the whole number of possible positions, is less than 0.29289 of the length of the line.

2. The number of positions in which the distance between the points is less than half the length of the line, is $\frac{3}{4}$ths of the whole number of possible positions.

3. The number of positions in which the distance between the points is less than one third of the length of the line, is $\frac{5}{9}$ths of the whole number of possible positions.

4. The number of positions in which the distance between the points is less than two thirds of the length of the line, is $\frac{8}{9}$ths of the whole number of possible positions.

5. If the line returns into itself like the circumference of the circle, the two points cannot be further apart than one half the length of the line, and for any given smaller distance, the number of positions, in which the distance between the points is less than this distance, is proportional to the distance.

6. If the line returns into itself, and if three points are assumed upon it, the number of positions, in which the two nearest points are at a less distance apart than one fourth of the length of the line, is $\frac{15}{16}$ths of the number of all possible positions.

7. In the preceding case, the least distance apart of the two nearest points is 0.09763 of the length of the line in one half of the whole number of possible cases.

8. In the preceding case, the three points are always included in an arc of the line, which is two thirds of the whole length;

in $\frac{3}{4}$ths of all the possible positions, they are included in half the length of the arc; the number of positions, in which the three points are included in an arc which is less than half the length, is proportional to the square of the length of the arc; the number of positions in which the three points are included in an arc which is $\frac{5}{8}$ths of the whole length is $\frac{63}{64}$ths of the whole number of positions.

VIRTUAL VELOCITIES.

BY WILLIAM WATSON.

It is proposed to deduce the law of virtual velocities from the general equations of equilibrium.

Let F_1, F_2, F_3, &c., be a number of forces by which any material system is held in equilibrium; f_1, f_2, f_3, &c., their respective lines of direction.

Suppose this system to receive any displacement whatever, δp, consistent with the conditions to which the system is subject, such that the component of δp due to the rotation of the system, is infinitesimal.

Then the algebraic sum of the products, obtained by multiplying the intensity of each force by the corresponding distance which its point of application advances in the direction of that force, must be equal to nothing; that is,

$$F_1 \delta f_1 + F_2 \delta f_2 + F_3 \delta f_3 + \&c. \ldots = 0;$$

which may be written $\quad \Sigma F \delta f = 0.$

The distance δf is called the virtual velocity with reference to the force, F.

The term velocity was first employed because the displacements, which were taken infinitely small, were conceived to be made in

the *same* infinitesimal time, and therefore the spaces were proportional to the velocities.

The term virtual was used to show that the displacements were hypothetical, and not actual. The term virtual moment is usually applied to the expression $F \delta f$.

The principle may be concisely stated thus; if any system is in equilibrium, the sum of the virtual moments of all the forces vanishes.

Let the components of δp be a translation δq, and a rotation $d\theta$ about any axis. This rotation is equivalent to three simultaneous rotations $d\theta_x$, $d\theta_y$, $d\theta_z$, about three rectangular axes X, Y, Z; therefore,
$$d\theta = \sqrt{(d\theta_x^2 + d\theta_y^2 + d\theta_z^2)}.$$

Resolving δq parallel to the axes X, Y, Z, and calling the respective components δq_x, δq_y, δq_z, we have
$$\delta q = \sqrt{(\delta q_x^2 + \delta q_y^2 + \delta q_z^2)}$$

Now find how much a point, whose coördinates are $(x\,y\,z)$, advances along the three axes of coördinates in virtue of the elementary rotation $d\theta$.

Let fall from $(x\,y\,z)$ a perpendicular ρ_z upon the axis Z. Then the angle, which the arc $\rho_z d\theta_z$ makes with X, is the same as that which ρ_z makes with Y, that is, $\cos^{-1}\frac{y}{\rho_z}$. Therefore, projecting the arc $\rho_z d\theta_z$ upon X and Y, and considering the rotation positive, we have

$$-\rho_z d\theta_z \frac{y}{\rho_z} = -y\,d\theta_z = \text{motion along } X \text{ in virtue of rotation } d\theta_z;$$
$$\rho_z d\theta_z \frac{x}{\rho_z} = x\,d\theta_z = \text{“ “ } Y \text{ “ “ } d\theta_z.$$

Similarly we find

$$-\rho_z d\theta_x \frac{z}{\rho_z} = -z\,d\theta_x = \text{“ “ } Y \text{ “ “ } d\theta_x;$$
$$\rho_z d\theta_x \frac{y}{\rho_z} = y\,d\theta_x = \text{“ “ } Z \text{ “ “ } d\theta_x;$$
$$-\rho_z d\theta_y \frac{x}{\rho_z} = -x\,d\theta_y = \text{“ “ } Z \text{ “ “ } d\theta_y;$$
$$\rho_z d\theta_y \frac{z}{\rho_z} = z\,d\theta_y = \text{“ “ } X \text{ “ “ } d\theta_y.$$

Combining these motions with the translations, and calling δx, δy, δz, the whole distance the point advances in the direction of X, Y, Z, we have

$$\delta x = \delta q_x + z\,d\theta_y - y\,d\theta_z, \qquad (a)$$
$$\delta y = \delta q_y + x\,d\theta_z - z\,d\theta_x, \qquad (b)$$
$$\delta z = \delta q_z + y\,d\theta_x - x\,d\theta_y, \qquad (c)$$

Now take the equations of equilibrium

$$\Sigma X = 0, \quad \Sigma Y = 0, \quad \Sigma Z = 0,$$
$$\Sigma(Zy - Yz) = 0, \quad \Sigma(Xz - Zx) = 0, \quad \Sigma(Yx - Xy) = 0,$$

and multiply them respectively by δq_x, δq_y, δq_z, $d\theta_x$, $d\theta_y$, $d\theta_z$; adding the products, we get

$$\Sigma\{X(\delta q_x + z\,d\theta_y - y\,d\theta_z) + Y(\delta q_y + x\,d\theta_z - z\,d\theta_x)$$
$$+ Z(\delta q_z + y\,d\theta_x - x\,d\theta_y)\} = 0. \qquad (d)$$

Using (a), (b), (c), we reduce (d) to

$$\Sigma\{X\delta x + Y\delta y + Z\delta z\} = 0. \qquad (e)$$

But $\quad \delta x = \delta p \cos{}^{x}_{\delta p}, \quad \delta y = \delta p \cos{}^{y}_{\delta p}, \quad \delta z = \delta p \cos{}^{z}_{\delta p},$

$$X = F\cos{}^{x}_{f}, \qquad Y = F\cos{}^{y}_{f}, \qquad Z = F\cos{}^{z}_{f};$$

substituting these values in (e) it becomes

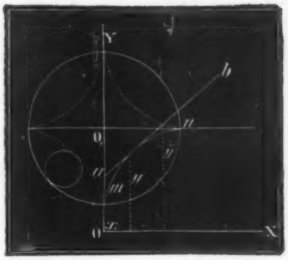
$$\Sigma\{F\delta p\,(\cos{}^{x}_{\delta p}\cos{}^{x}_{f} + \cos{}^{y}_{\delta p}\cos{}^{y}_{f} + \cos{}^{z}_{\delta p}\cos{}^{z}_{f})\} = 0;$$

or $\quad \Sigma\{F\delta p\cos{}^{f}_{\delta p}\} = \Sigma F\delta f = 0.$

PRACTICAL APPLICATION.

The principle of Virtual Velocities may be applied with advantage to the solution of the following problem, which was first proposed by WALTON.

A heavy rod $a\,b$, situated in the plane $X\,O\,Y$, rests with one end against a smooth vertical wall $O\,Y$, and upon a smooth plane curve, $m\,n$; to determine the nature of the curve that the rod may be in equilibrium in every position.



Let $2l =$ length of rod,

$(xy) =$ coördinates of the point of tangency,

$\bar{y} =$ ordinate of centre of gravity of the rod,

$$\bar{y} = y + (l - xD_x s)\, D_s y$$
$$= y - xD_x y + l D_s y.$$

If the rod is displaced, $\bar{y}$ becomes $\bar{y} + d\bar{y}$; but by the principle of virtual velocities $F d\bar{y} = 0$ for equilibrium, F being the weight of the rod. $\therefore \quad d\bar{y} = 0 = -\, xD_x^2 y + D_x l \frac{D_x y}{1 + (D_x y)^2}$

$$= -\, xD_x^2 y + \frac{l D_x^2 y}{(1 + (D_x y)^2)^{\frac{3}{2}}}.$$

This equation is satisfied by putting $D_x^2 y = 0$, which is the differential equation of a straight line; also by putting

$$x(1 + (D_x y)^2)^{\frac{3}{2}} - l = 0; \text{ from which } D_x y = \sqrt{(l^{\frac{2}{3}} - x^{\frac{2}{3}})}\,.\,x^{-\frac{1}{3}}$$
$$\therefore\ y = \int \sqrt{(l^{\frac{2}{3}} - x^{\frac{2}{3}})}\; x^{-\frac{1}{3}} dx + \text{const.} = -\,(l^{\frac{2}{3}} - x^{\frac{2}{3}})^{\frac{3}{2}} + \text{const.}$$

If we take for the axis of x the horizontal position of the rod, then the constant disappears, and we have, after reducing, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = l^{\frac{2}{3}}$, which is the equation of a hypocycloid, in which l is the radius of the fixed circle, and $\frac{l}{4}$ the radius of the moving circle.

THE PRISMOIDAL FORMULA.

BY CHAUNCEY WRIGHT.

The formula $(B + 4B' + B'')\frac{l}{6}$, in which B, B', B'' are three equidistant parallel surfaces, sections of a solid, and l the distance between B and B'', is the expression for the solid contents between

B and B'', not only for the prismoid from which it was first derived, but also for several solids of revolution, as the sphere, ellipsoid, &c. What is the extent of its application?

Let X be the axis perpendicular to which the sections are made, and $f(x)$ the area of the section at the distance x from the origin. The problem then is, What function f will fulfil the conditions of the formula?

Let three sections be made through any solid at the distances $(x - h)$, x, $(x + h)$ from the origin. Then $h = \frac{l}{2}$; and $f(x - h)$, $f(x)$, $f(x - h)$ will be the areas of the sections, and if the formula apply the solid contents between $f(x - h)$ and $f(x + h)$ will be $[f(x - h) + 4f(x) + f(x + h)]\frac{h}{3}$. But the solid contents is also equal to the integral of the differential solid $f(x)\,dx$ between the limits $x - h$ and $x + h$.

Equating these two expressions for the solid contents, we have, if the function f fulfils the conditions of the formula,

$$\int_{x-h}^{x+h} f(x)\,dx = \int f(x + h)\,dx - \int f(x - h)\,dx$$
$$= \Big[f(x - h) + 4f(x) + f(x + h)\Big]\frac{h}{3}.$$

To find what form of f will satisfy this equation, develop both its members by Taylor's theorem. The first member becomes,

$$(A)\;\left.\begin{aligned} &\int\Big[f(x) + f'(x)h + f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{2\cdot 3} + \&c.\Big]\,dx \\ -&\int\Big[f(x) - f'(x)h + f''(x)\frac{h^2}{2} - f'''(x)\frac{h^3}{2\cdot 3} + \&c.\Big]\,dx \end{aligned}\right\}$$
$$= \int 2f'(x)h\,dx + \int 2f'''(x)\frac{h^3}{2\cdot 3}\,dx + \&c.$$
$$= 2f(x)h + f''(x)\frac{h^3}{3} + \overset{\text{IV}}{f}(x)\frac{h^5}{3\cdot 4\cdot 5} + \&c.$$

The second member becomes

$$(B)\quad \left.\begin{array}{l}\Big[f(x)-f'(x)\,h+f''(x)\frac{h^2}{1.2}-\text{\&c.}+4\,f(x)\\ +f(x)+f'(x)\,h+f''(x)\frac{h^2}{1.2}+\text{\&c.}\Big]\frac{h}{3}\end{array}\right\}$$

$$=\Big[6f(x)+f''(x)\,h^2+\overset{\text{IV}}{f}(x)\frac{h^4}{3.4}+\text{\&c.}\Big]\frac{h}{3}$$

$$=2f(x)\,h+f''(x)\frac{h^3}{3}+\overset{\text{IV}}{f}(x)\frac{h^5}{36}+\text{\&c.}$$

Comparing the last members of (A) and (B) we find the equation

$$2f(x)\,h+f''(x)\frac{h^3}{3}+\overset{\text{IV}}{f}(x)\frac{h^5}{60}+\text{\&c.}=2f(x)\,h+f''(x)\frac{h^3}{3}$$
$$+\overset{\text{IV}}{f}(x)\frac{h^5}{36}+\text{\&c.}$$

which is satisfied only when the terms beyond the second disappear; that is, by functions which have no fourth and higher derivatives. Hence $f(x)$ must be an algebraic expression of positive integral powers not exceeding the third degree.

In general

$$f(x)=a\,x^3+b\,x^2+c\,x+e=\text{area of section.}$$

The error in applying this formula, when $f(x)$ is of the fourth or fifth degree is

$$\overset{\text{IV}}{f}(x)\,h^5\left(\tfrac{1}{36}-\tfrac{1}{60}\right)=\overset{\text{IV}}{f}(x)\frac{h^5}{90}.$$

If h is taken so small, that terms containing its fifth power may be neglected, the prismoidal formula may be applied to any solid. Simpson's rule, which includes a series of prismoidal formulas, is sufficiently accurate when n is so large that $\left(\frac{l}{n}\right)^5$ may be neglected.

The prismoid comes within the limits of this formula. For, being a solid bounded by planes, and the sections perpendicular to its sides being dissimilar polygons, but with the same number of sides, it may be decomposed into prismoids with triangular sections. These prismoids are bounded by one plane and two warped surfaces; the plane

surface being one of the surfaces of the original prismoid; the other two being generated by the varying sides of the triangular section. Since these triangles are not similar, their sides must change at dissimilar rates in passing along the axis, or their base and altitude will not change proportionally.

Let b be the base and a the altitude of any triangle at the origin of x; let α and β be their respective rates of change, in passing along x. Then $b + \beta x$ and $a + \alpha x$ will be the base and altitude at any distance x from the origin, and its area will be $\frac{1}{2}(a + \alpha x)(b + \beta x)$. Hence the area of the whole polygonal section is
$f(x) = \frac{1}{2}\Sigma(a + \alpha x)(b + \beta x) = \frac{1}{2}\Sigma\,[a b + (a\beta + b\alpha)x + \alpha\beta x^2]$
which is an expression of the second degree, and is therefore one to which the formula applies.

The formula also applies to the solids generated by the revolution of the conic sections about their axes of symmetry, the general equation of these sections being $y^2 = m x + n x^2$. The sections of these solids taken perpendicularly to the axis of x are circles, having y for a radius, and $\pi m x + \pi n x^2 = \pi y^2 = f(x) =$ area, to which also the formula applies.

But if the conic section be revolved about any axis for which the general equation does not hold, then the prismoidal formula does not apply, because the expression for the area of the section $f(x)$ will involve radicals which have fourth and all higher derivatives. The formula applies in like manner to the solid of revolution of the semicubic parabola, of which the equation is $y^2 = m x^3$.

The principles involved in this discussion of the prismoidal formula, may be applied to finding a formula, for solid contents, which shall hold good for bodies, of which the sections are expressed in terms of higher degrees than the third; and also for cases in which the sections are taken at unequal intervals.

This discussion is reserved for the next number of the Monthly.

OVALS AND THREE-CENTRE ARCHES.

BY J. B. HENCK.

An oval, $ABCD$ (fig. 1) is composed of two pairs of circular arcs, of which one pair EF and GH have their centres on the diameter BD, and the other pair EG and FH have their centres on the diameter AC or on this diameter produced, the two diameters being at right angles to each other. A three-centre arch is one half, BAD, of an oval; so that the half oval only need be considered.

Fig. 1.

As these figures are of importance to the engineer and architect, it is proposed to give a general method of drawing the oval, when its two diameters are given, and one of the radii is assumed. A general algebraic relation between the semidiameters and the radii will also be established. The special cases derived from this general relation will serve to correct some erroneous methods of drawing ovals given in drawing books, and will also give the foundation of such methods as are correct, and the means of *computing* the radii, matters generally neglected.

GEOMETRICAL METHOD, WHEN ONE RADIUS IS ASSUMED.

Let BD and AC (fig. 1) be the diameters of the required oval, and let BI be assumed as the first radius. From I as a centre describe the quadrant BEK. Through A and K draw a straight line, and produce it, till it meets the curve AK in E. Through E and I draw the line EL, cutting AC, produced if necessary in L. Then L is the centre of the second arc of the oval, and E is the common point of tangency of the two arcs. To prove this, it will be suffi-

cient to show that $AL = EL$. Draw KI, and we have the triangle AEL similar to the isosceles triangle KEI. Therefore, $AL = EL$.

If the radius AL had been assumed, and with it, the quadrant AEM described, the point E would have been found by drawing a straight line through M and B, and producing it till it met the arc AM.

It should be observed, that the first radius must always be less than AN, and the second radius always greater than BN.

GENERAL RELATION BETWEEN THE SEMI-DIAMETERS AND THE RADII.

Let $BN = a$ (fig. 1), $AN = b$, $BI = r$ and $AL = r_1$. Then in the right triangle INL we have $IL = r_1 - r$, $IN = a - r$ and $NL = r_1 - b$.

$$\therefore (r_1 - r)^2 = (a - r)^2 + (r_1 - b)^2,$$

$$\therefore r_1^2 - 2rr_1 + r^2 = a^2 - 2ar + r^2 + r_1^2 - 2br_1 + b^2,$$

$$\therefore 2ar + 2br_1 - 2rr_1 = a^2 + b^2. \qquad (1)$$

This equation expresses the relation that must always unite the four quantities a, b, r, and r_1. When any three of these are given, the fourth may be found. When a and b are considered fixed, the value of either radius may be assumed, and the other calculated. Thus if the first radius r be assumed, (1) gives for the second radius

$$r_1 = \frac{a^2 + b^2 - 2ar}{2b - 2r} = a + \frac{(a - b)^2}{2b - 2r}; \qquad (2)$$

and if the second radius r_1 be assumed, (1) gives for the first radius

$$r = \frac{a^2 + b^2 - 2br_1}{2a - 2r_1} = b - \frac{(a - b)^2}{2r_1 - 2a} \qquad (3)$$

Example. If one diameter of an oval be 24, and the other 18, and if the first radius be assumed equal to 6, we have $a = 12$, $b = 9$, and $r = 6$, to find r_1. Here (2) gives for the second radius

$$r_1 = 12 + \frac{3^2}{18 - 2 \times 6} = 12 + \tfrac{9}{6} = 13.5.$$

If the second radius had been assumed $= 13.5$, the first radius would be from (3)

$$r = 9 - \frac{3^2}{2 \times 13.5 - 24} = 9 - \tfrac{9}{3} = 6.$$

The applications are reserved for the next number.

Mathematical Monthly Notices.

The Quarterly Journal of Pure and Applied Mathematics. Edited by J. J. SYLVESTER, *M. A., F. R. S., late Professor of Natural Philosophy in University College, London; and* N. M. FERRERS, *M. A., Fellow of Gonville and Caius College, Cambridge: assisted by* G. G. STOKES, *M. A., F. R. S., Lucasian Professor of Mathematics in the University of Cambridge;* A. CAYLEY, *M. A., F. R. S., late Fellow of Trinity College, Cambridge; and* M. HERMITE, *Corresponding Editor in Paris.* London: JOHN W. PARKER AND SON, West Strand.

THIS journal may be considered as the continuation of the Cambridge and Dublin Mathematical Journal, the publication of which was discontinued with the ninth volume in 1854. The first number of the Quarterly Journal bears date April, 1855, and number eight, which completes the second volume, was published May, 1858. The next number will appear in October. We shall only say at present, that this Journal is an able exponent of the present state and progress of mathematical science in England.

Nouvelles Annales de Mathématiques. Journal des Candidats aux écoles Polytechnique et Normale: Rédigé par M. TERQUEM, *Officier de l'Université, Docteur des Sciences, Professeur aux Écoles Impériales d'Artillerie, Officier de la Légion d'honneur, et* M. GERONO, *Professeur des Mathématiques.* Paris: MALLET-BACHLIER.

This journal is issued on the first of every month, containing either 32 or 48 pages, 8vo. It was begun in 1842, and sixteen volumes are now complete. The last number received is for July, 1858. In 1855 the "Bulletin de Bibliographie, d'Histoire et de Biographie Mathématiques" was commenced, a few pages being added to each number of the journal. The "Bulletin" is paged independently, and makes an annual volume of about 200 pages, of which three are complete. This journal contains solutions and discussions of all degrees of difficulty, but its chief aim evidently is to influence the course of mathematical study, and stimulate both teachers and students to aim at a high standard of excellençe; and from the character of the journal we have no doubt of the result.

Journal für die reine und angewandte Mathematik. In zwanglosen Heften. Als Fortsetzung des von A. C. CRELLE, *gegründeten Journals herausgegeben unter Mitwirkung der* HERREN STEINER, SCHELLBACH, KUMMER, KRONECKER, WEIRSTRASS, *von* C. W. BORCHARDT.

Mit thätiger Beförderung hoher Königlich-Preufsischer Behörden. **Berlin: Druck und Verlag von Georg Reimer.**

Crelle's Journal was begun in 1826, and has been continued without interruption until the present time. Each volume contains from 375 to 400 quarto pages; the one for 1857 being the fifty-third of the series. The fiftieth volume, issued in 1855, contains a table of contents of the fifty volumes, and an examination of this table must convince any one of the great value of this journal. We find the papers of Abel, 24 in number, making 384 pages; 19 by Cayley, pp. 216; 44 by Crelle, pp. 1187; 26 by Lejeunne-Dirichlet, pp. 390; 37 by Eisenstein, pp. 698; 38 by Gudermann, pp. 1489; 25 by Hesse, pp. 329; 98 by Jacobi, pp. 1575, besides posthumous papers still appearing; 25 by Kummer, pp. 523; 29 by Minding, pp. 216; 26 by Möbius, pp. 361; 6 by Ottinger, pp. 1035; 12 by Olivier, pp. 121; 6 by Plana, pp. 293; 21 by Plücker, pp. 346; 37 by Raabe, pp. 413; 22 by Richelot, pp. 533; 13 by Schellbach, pp. 197; 51 by Steiner, pp. 687; 28 by Stern, pp. 400; with over 200 other contributors. In short, it would be difficult to exaggerate the value of this Journal, and we believe that no one could enter upon any investigation in the higher mathematics, before consulting Crelle, without running a great risk of having been anticipated in its pages. We sincerely hope that Professors of Mathematics and all Library Committees will remember that this journal is the very best possible foundation for a mathematical library, which cannot be complete without it.

Special notices of the labors of various contributors to the above journals are reserved for future occasions.

Physical and Celestial Mechanics. By Benjamin Peirce, *Perkins Professor of Astronomy and Mathematics in Harvard University, and consulting Astronomer of the American Ephemeris and Nautical Almanac. Developed in four systems of Analytic Mechanics, Celestial Mechanics, Potential Physics, and Analytic Morphology.* Boston: Little, Brown & Co. 1855.

Of this proposed course on Mechanics, the first volume, entitled A System of Analytic Mechanics, is already issued. Some twelve years ago, Prof. Peirce announced a series of three volumes, on "Curves, Functions, and Forces." Volumes I. and II., known by the name of "Curves and Functions," were then published by James Munroe & Co.; and the volume before us, although a part of a new and vastly enlarged course, may be considered as the redeeming of this former pledge.

It is unnecessary for us to say that this is Prof. Peirce's greatest work, and more than realizes our highest expectations.

Every part of the work is treated with that concise fulness for which the author is so remarkable, and which has enabled him to condense more matter into 496 quarto pages than was ever before put into the same space. In fact, we do not think of a single subject, coming strictly within the limits of the first volume of the course, left untouched. Some idea may be given of the amount the volume contains, by remarking that the table of contents covers twenty-seven closely printed double-column pages, and the alphabetical index, ten similar pages. We need not be more specific, for mathematicians will read and judge of the work for themselves. Besides, we shall have occasion to refer to its contents hereafter.

A striking feature of the volume is the style in which it is issued. We unhesitatingly pronounce it the finest specimen of mathematical printing we have ever seen, and do not believe it has ever been surpassed, if equalled, in any country. For this, the publishers deserve and receive the greatest credit. It is but justice, however, and it gives us great pleasure to add, that the work was printed by Messrs Allen & Farnham, of Cambridge, and our readers will not fail to discover the evidence of their skill in the Mathematical Monthly.

SCIENTIFIC BOOKS.

HUTTON. Mathematical Tables. 8vo. $3.75.
HYMER. Plane and Spherical Trigonometry. $2.50.
———. Algebraic Equations. 8vo. $3.00.
———. Analytical Geometry. $3.00.
———. Differential Equations, and Calculus of Finite Differences. 8vo. $3.60.
———. Integral Calculus. 8vo. $3.00.
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